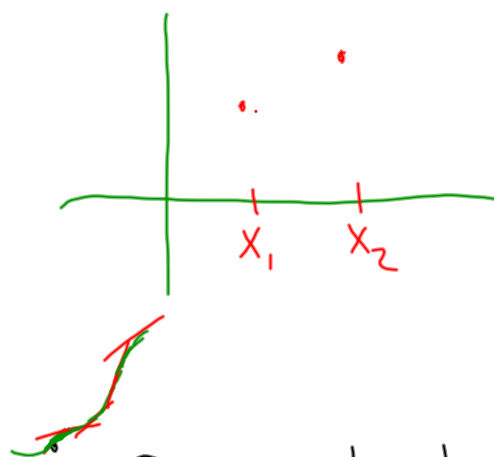


5.1  
 How do you define things,  
 And what properties are equivalent?

Ex. A function is increasing on an interval, if

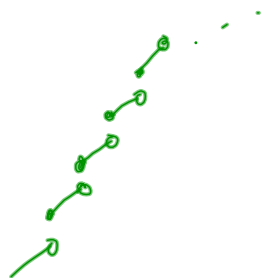
DEF

for every pair of numbers  $x_1$  and  $x_2$ ,  
 when  $x_1 < x_2$  then  $f(x_1) < f(x_2)$ .



Calculus "view".  
 If a function is continuous  
 on  $[a, b]$  and differentiable  
 then on  $(a, b)$  then  
 if, whenever  $x$  in  $(a, b)$   
 $f'(x) > 0$  then  
 $f(x)$  is increasing.

Positive derivative  $\Rightarrow$  increasing  
 original  
 function

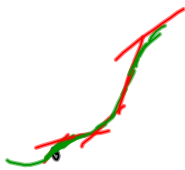
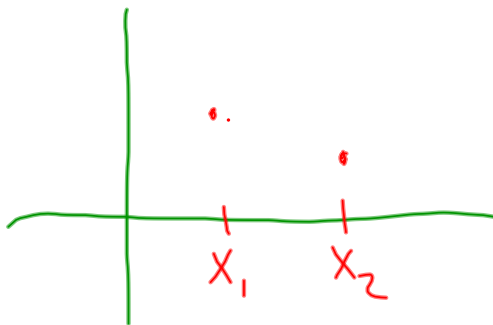


5.1 How do you define things,  
And what properties are equivalent?

Ex. A function is decreasing on an interval, if

DEF

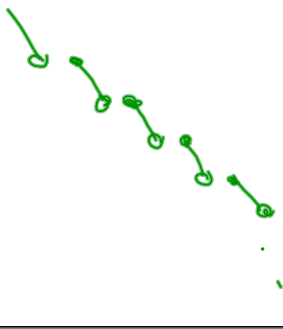
for every pair of numbers  $x_1$  and  $x_2$ ,  
when  $x_1 < x_2$  then  $f(x_1) > f(x_2)$ .



Calculus "view".

If a function is continuous  
on  $[a, b]$  and differentiable  
then on  $(a, b)$  then  
if, whenever  $x$  in  $(a, b)$   
 $f'(x) < 0$  then  
 $f(x)$  is increasing.

negative derivative  $\Rightarrow$  decreasing  
original  
function



A popular alternative definition

A  $f^n \dots$  is increasing

If, for every  $x_1, x_2$

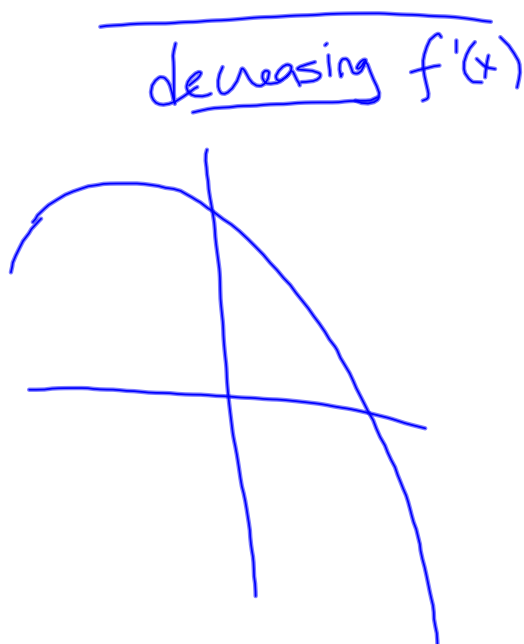
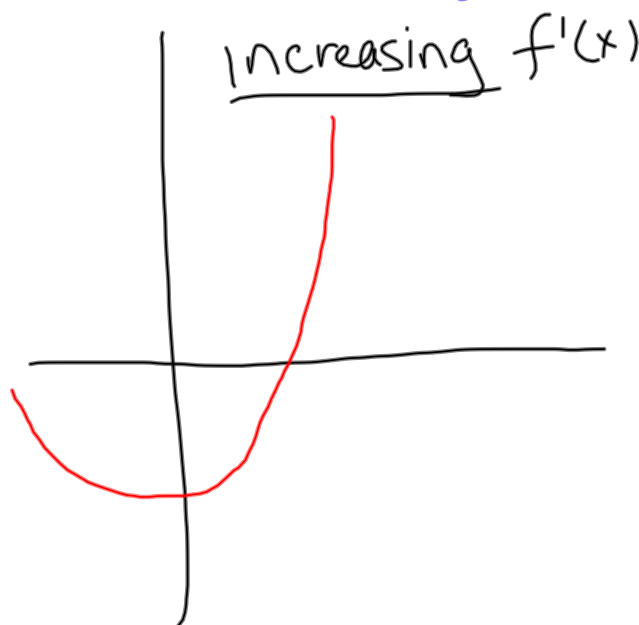
$$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$$

What if they want to talk about  $f^n$ , that  
satisfy  $f(x_1) < f(x_2)$ ?  
they call this **STRICTLY** increasing.

Positive derivative  $\Rightarrow$  increasing  
original  
function

Negative derivative  $\Rightarrow$  decreasing  
original  
function.

- now, figure out what increasing derivatives do.
- start with positive derivative
- start with negative derivative



Definition If  $f(x)$  is differentiable on  $(a, b)$   
 and if  $f'(x)$  is INCREASING on  $(a, b)$   
 then  $f(x)$  is CONCAVE UP



... DECREASING  $f'(x)$   
 $\Rightarrow$  CONCAVE DOWN



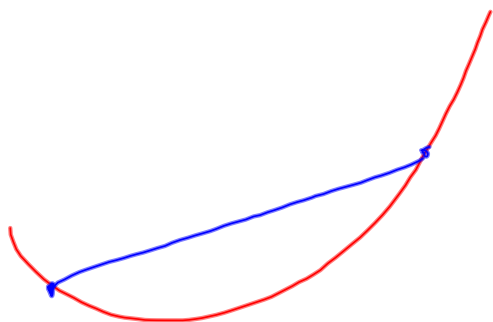
"EQUIVALENT" Property

if  $f'(x)$  is INCREASING, what do we know  
 about its derivative?

If  $f''(x) > 0$ , then  $f(x)$  is concave up.

A function is concave up <sup>on an interval</sup> if

whenever I connect two points on that curve, the line is above the curve



A point of inflection is a point on the  
curve where concavity changes

[[from neg  $\rightarrow$  pos, or pos  $\rightarrow$  neg]]